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**IDX G9 MATH H/H+ STUDY GUIDE ISSUE 3**

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3.1 Concepts and Properties of Polygons

Polygon: closed, planar figure with ≥ 3 straight sides such that no three adjacent points are collinear and edges only intersect at common vertices.

Diagonal: a segment that connects two non-adjacent points in a polygon

|  |  |
| --- | --- |
| #Sides | Name |
| 3 | triangle |
| 4 | quadrilateral |
| 5 | pentagon |
| 6 | hexagon |
| 7 | heptagon |
| 8 | octagon |
| 9 | nonagon |
| 10 | decagon |
| 11 | hendecagon |
| 12 | dodecagon |
| n | n-gon |

Special polygons:

-Equilateral: All sides are congruent in length

-Equiangular: All angles are congruent

-Regular: Both Equilateral and Equiangular

Convex polygons: polygons which have all diagonals inside

Concave polygons: polygons that are not convex

How to determine whether a polygon is convex: check if any angle > 180; if so, it is concave; otherwise, it is convex.

All polygons are assumed to be convex in problems unless otherwise stated.

3.2 Parallelograms

Parallelogram: Quadrilateral with both pairs of opposite sides parallel (and equal)

Definition and formulas: see back

How to quickly construct an arbitrary parallelogram:

1. Draw a line k.
2. Draw another line segment m, intersecting line k at A and of some length n.
3. Copy angle A on somewhere else on k.
4. Copy the length n.
5. Connect the endpoints.

3.3 Special Parallelograms

1. Rectangle: A parallelogram with 90° angles

2. Rhombus: A parallelogram with equal-length sides

3. Square: A parallelogram with 90° angles and equal-length sides

Special properties:

A rectangle has congruent diagonals.

A rhombus has perpendicular diagonals (so its area formula is 1/2 \* d1 \* d2, where d1and d2 are the lengths of its diagonals)

A square has congruent, perpendicular diagonals whose lengths are equal to \* s, where s is the square's side length.

3.4 Trapezoids

A trapezoid is a quadrilateral with exactly one pair of parallel sides (called bases); the non-parallel sides are its legs.

Area formula: (a + b) \* h/2, where a and b are the lengths of the bases, and h is the height of the trapezoid (the distance between the bases).

Solving trapezoid problems:

1. Cut into a right triangle and a right trapezoid if it is isosceles.
2. Cut into a triangle and a parallelogram.
3. Cut along altitudes from the upper base to the lower base to get two RTs and a rectangle.
4. Construct parallel lines to the legs (basically 2).
5. Extend the legs to intersect at a point (very useful for isosceles trapezoids)

Special trapezoids:

1. Right trapezoid: A trapezoid with a right angle.
2. Isosceles trapezoid: A trapezoid with congruent leg lengths.

Midsegment: a segment connecting the midpoint of the two legs.

4.1 Ratios and Proportions

Ratio: comparison of two quantities

Expressed as a : b or

The one ratio to note: Golden ratio

4.2 Similar Triangles

If ΔABC ~ ΔDEF, then ∠A = ∠D, ∠B = ∠E, ∠C = ∠F,

Formulas:

Polygon interior angle-sum theorem:

Sum of interior angles of an n-gon is (n – 2) \* 180°

Polygon exterior angle-sum theorem:

Sum of exterior angles of an n-gon is 360°

Parallelogram theorems:

Property theorem 1 (definition): If a quadrilateral ABCD is a parallelogram, then AB || CD, AD || BC.

Property theorem 2: If ABCD is a parallelogram, then AB = CD, AD = BC.

Property theorem 3: If ABCD is a parallelogram, then ∠A = ∠C, ∠B = ∠D.

Property theorem 4: If ABCD is a parallelogram, then its diagonals bisect each other.

Determination theorem 1 (definition): If AB || CD, AD || BC, then quadrilateral ABCD is a parallelogram.

Determination theorem 2: If AB = CD, AD = BC, then ABCD is a parallelogram.

Determination theorem 3: If AB = CD, AB || CD, then ABCD is a parallelogram.

Determination theorem 4: If ∠A = ∠C, ∠B = ∠D, then ABCD is a parallelogram.

Determination theorem 5: If AC, BD bisect each other, the ABCD is a parallelogram.

Rectangles:

Property theorem 1 (definition): If quadrilateral ABCD is a rectangle, then ∠A = ∠B = ∠C = ∠D = 90°.

Property theorem 2: If ABCD is a rectangle, then AC = BD.

Determination theorem 1 (definition): If ∠A = ∠B = ∠C = 90°, then ABCD is a rectangle.

Determination theorem 2: In ▱ABCD, if AC = BD, then ABCD is a rectangle.

Rhombi:

Property theorem 1 (definition): If ABCD is a rhombus, then AB = BC = CD = DA.

Property theorem 2: If ABCD is a rhombus, then AC ⊥ BD, ∠BAO = ∠DAO, ∠ABO = ∠CBO, ∠BCO = ∠DCO, ∠CDO = ∠ADO.

Determination theorem 1 (definition): If AB = BC = CD = DA, then ABCD is a rhombus.

Determination theorem 2: In ▱ABCD, if AC ⊥ BD, then ABCD is a rhombus.

Squares:

Property theorem 1 (definition): If ABCD is a square, ∠A = ∠B = ∠C = ∠D = 90° and AB = BC = CD = DA.

Property theorem 2: If ABCD is a square, then AC, BD are perpendicular bisectors, and they bisect their corresponding interior angles.

Determination theorem 1: In rectangle ABCD, if AB = BC, then ABCD is a square.

Determination theorem 2: In rhombus ABCD, if ∠A = 90°, then ABCD is a square.

Trapezoids:

Property theorem 1 (definition): If ABCD is an isosceles trapezoid where AD || BC, then AB = CD.

Property theorem 2: In isosceles trapezoid ABCD, if AB = CD, then ∠A = ∠D, ∠B = ∠C.

Property theorem 3: In isosceles trapezoid ABCD, if AB = CD, then AC = DB.

Determination theorem 1 (definition): In trapezoid ABCD where AD || BC, if AB = CD, then ABCD is an isosceles trapezoid.

Determination theorem 2: In trapezoid ABCD, AD || BC. If ∠B = ∠C, then ABCD is an isosceles trapezoid.

Determination theorem 3: In trapezoid ABCD with AD || BC, if AC = BD, then ABCD is an isosceles trapezoid.

Trapezoid midsegment theorem: In trapezoid ABCD with AD || BC, if E, F are midpoints of AB, CD respectively, then EF || AD || BC, EF = (AD + BC) / 2.

Proportion properties:

1. If , then ad = bc.
2. If , then .
3. If , then .
4. If , then .

Golden Ratio:  ≈ 1.618, ≈ 0.618

Side-splitter theorem: If a line parallel to one side of a triangle intersects the lines containing the other two sides, then the line divides the segments proportionally.

Corollary: If a line is parallel to one side of a triangle and intersects the lines containing the other two sides, then it divides the triangle's three sides proportionally.

Similar triangle determination:

1. If line l || BC, l ∩ AC = D, l ∩ AC = E, then ΔABC ~ ΔADE.
2. AA~: If ∠A = ∠D, ∠B = ∠E, then ΔABC ~ ΔDEF.
3. SAS: If ∠A = ∠D, , then ΔABC ~ ΔDEF.
4. SSS: If , then ΔABC ~ ΔDEF.
5. HL: For Rt ABC, Rt DEF, ∠C = ∠F = 90°, if , then ΔABC ~ ΔDEF.

Euclidean Theorem: In Rt ABC with ∠C = 90°, if CD ⊥ AB at D, then CD2 = AD \* BD, AC2 = AD \* AB, BC2 = BD \* AB.

Triangle interior angle bisector theorem: In ΔABC, if AD bisects ∠A, AD ∩ BC = D, then (with converse).

Triangle exterior angle bisector theorem: In ΔABC, if AD bisects the exterior angle of ∠A, AD ∩ BC = D, then (with converse).